# Observation on obtain infinitely many Pythagorean triangles

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**Abstract—** This paper presents The binary quadratic equation 11 1 2 2 y = x + is analyzed for its distinct integer solutions and we obtain infinitely many Pythagorean triangles. A few interesting relations among the sides are also given.

**Keywords:** *The binary quadratic equation, Pell equation,Pythagorean triangles, Integer Solutions.*

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## I. INTRODUCTION

Number theory is a branch of pure mathematics dedicated mainly to the study of the integers. Mathematics is the queen of the sciences and number theory is the queen of mathematics. The Diophantine equation is of the form 1 2 2 x = dy + where d a nonsquare positive integer is and integer solutions sought for x and y. This equation was first studied in India, starting with Brahmagupta, who developed the Chakravala method to solve Pell's equation. Pell's equation has infinitely many distinct integer solutions when 'd ' takes different numerical values. In this communication, by applying the non-trivial integral solutions of the binary quadratic equation 11 1 2 2 y = x + , we obtain infinitely many Pythagorean triangles. The recurrence relations satisfied by the sides of the triangle are presented.

## II. METHOD OF ANALYSIS

Consider the binary quadratic equation 11 1 2 2 y = x + whose general solution ( ) n n x , y is represented by ( ) ( ) (10 3 11) (10 3 11) , 0,1,2,3,K 2 1 10 3 11 10 3 11 2 11 1 1 1 1 1 = ⌊⌋⌈ ⌊⌈ = + + − ⌊⌋⌈ ⌊⌈ = + − − + + + + y n x n n n n n n Assuming n bn x = and n an bn y 3 10 = − , we obtain ( ) ( ) ⌊⌋⌈ ⌊⌈ = + − − +2 +2 10 3 11 10 3 11 6 11 1 n n an ( ) ( ) ⌊⌋⌈ ⌊⌈ = + − − +1 +1 10 3 11 10 3 11 2 11 1 n n bn , where n = 0,1,2,3,K Considering an bn , as the generators of a Pythagorean triangle, its legs Xn Yn , and hypotenuse Zn are found to be ( )( )( )( )( )( )( )( )( )( )( )( )( ) ⌊⌋⌈ ⌊⌈ = + + + − − − ⌊⌋⌈ ⌊⌈ = + + + − − + ⌊⌋⌈ ⌊⌈ = = + + + − − − 10 3 11 80992 24420 11 10 3 11 80992 24420 11 20 396 1 10 3 11 77410 23340 11 10 3 11 77410 23340 11 16 396 1 10 3 11 3970 1197 11 10 3 11 3970 1197 11 20 66 1 2 2 2 2 2 2 2 n n n n n n n n n n Z Y X a b The above values of Xn

Yn , and Zn satisfy the following recurrence relations: 398 20 398 16 398 80 2 1 2 1 2 1 − + = − + = − − + = + + + + + n n n n n n n n n Z Z Z Y Y Y X X X The following table shows a few numerical Pythagorean triangles: Table 1. Numerical Examples

$$y^2 = 11x^2 + 1$$

whose general solution $(x_n, y_n)$ is represented by

$$x_n = \frac{1}{2\sqrt{11}}\left[\left(10+3\sqrt{11}\right)^{n+1} - \left(10-3\sqrt{11}\right)^{n+1}\right]$$

$$y_n = \frac{1}{2}\left[\left(10+3\sqrt{11}\right)^{n+1} + \left(10-3\sqrt{11}\right)^{n+1}\right], \quad n = 0,1,2,\ldots$$

Assuming $x_n = b_n$ and $y_n = a_n - \frac{10}{3}b_n$, we obtain

$$a_n = \frac{1}{6\sqrt{11}}\left[\left(10+3\sqrt{11}\right)^{n+2} - \left(10-3\sqrt{11}\right)^{n+2}\right]$$

$$b_n = \frac{1}{2\sqrt{11}}\left[\left(10+3\sqrt{11}\right)^{n+1} - \left(10-3\sqrt{11}\right)^{n+1}\right],$$

Considering $a_n, b_n$ as the generators of a Pythagorean triangle, its legs $X_n, Y_n$ and hypotenuse $Z_n$ are found to be

$$X_n = 2a_n b_n = \frac{1}{66}\left[\left(10+3\sqrt{11}\right)^{2n}\left(3970+1197\sqrt{11}\right)+\left(10-3\sqrt{11}\right)^{2n}\left(3970-1197\sqrt{11}\right)-20\right]$$

$$Y_n = \frac{1}{396}\left[\left(10+3\sqrt{11}\right)^{2n}\left(77410+23340\sqrt{11}\right)+\left(10-3\sqrt{11}\right)^{2n}\left(77410-23340\sqrt{11}\right)+16\right]$$

$$Z_n = \frac{1}{396}\left[\left(10+3\sqrt{11}\right)^{2n}\left(80992+24420\sqrt{11}\right)+\left(10-3\sqrt{11}\right)^{2n}\left(80992-24420\sqrt{11}\right)-20\right]$$

The above values of $X_n, Y_n$ and $Z_n$ satisfy the following recurrence relations:

$$X_{n+2} - 398X_{n+1} + X_n = 80$$
$$Y_{n+2} - 398Y_{n+1} + Y_n = -16$$
$$Z_{n+2} - 398Z_{n+1} + Z_n = 20$$

### Table 1. Numerical Examples

| $n$ | $X_n$ | $Y_n$ | $Z_n$ |
|---|---|---|---|
| 1 | 47880 | 155601 | 162801 |
| 2 | 19056240 | 61928791 | 64794409 |
| 3 | 7584335760 | 24647503200 | 25788012000 |

### III. CONCLUSION

In this paper, we have presented infinitely many Pythagorean triangles for the considered binary quadratic equation 11 1 2 2 y = x + . As the binary quadratic equations are rich in variety, one may search for the other solutions of the considered binary quadratic equations and determine their integer solutions along with suitable properties.

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